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CORRELATION OF THEORY AND EXPERIMENT ON

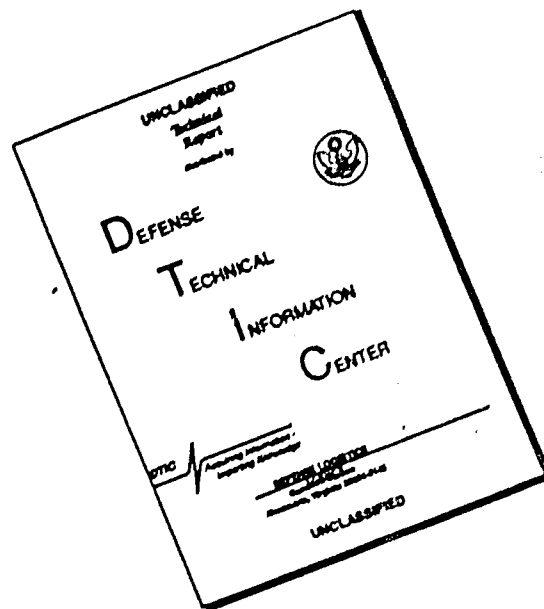
STAGNATION POINT HEAT TRANSFER RATES TO

CYLINDERS AND SPHERES

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CORRELATION OF THEORY AND EXPERIMENT ON

STAGNATION POINT HEAT TRANSFER RATES TO

CYLINDERS AND SPHERES

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R.P. Hawerton  
3-28-59

PREPARED BY

*J. Vasilu*  
J. Vasilu

CHECKED BY

*J. M. Bowyer, Jr.*  
J. M. Bowyer, Jr.

APPROVED BY

*W. F. Radcliffe*  
W. F. Radcliffe  
Chief Development Engineer

APPROVED BY

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POINT HEAT TRANSFER RATES TO CYLINDERS AND SPHERES. (U)

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DESCRIPTORS: (ROCKET MOTORS (LIQUID PROPELLANT),  
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VISCOSITY, INCOMPRESSIBLE FLOW, CORRELATION  
TECHNIQUES  
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(U)

THE REPORT PRESENTS A COMPARISON BETWEEN A  
SIMPLIFIED EQUATION FOR STAGNATION POINT HEAT  
TRANSFER AND THE EXPERIMENTAL RESULTS OF AN  
INVESTIGATION ON HEATING RATES TO CYLINDERS AND  
SPHERES SUBJECTED TO FLAME IMPINGEMENT FROM A 1/10-  
SCALE MODEL OF AN ATLAS VERNIER ENGINE, CARRIED OUT  
UNDER CONTRACT NO. AF-04(645)-4.

(AUTHOR)

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FORWARD

This report presents a comparison between a simplified equation for stagnation point heat transfer and the experimental results of an investigation on heating rates to cylinders and spheres subjected to flame impingement from a 1/10-scale model of an Atlas vernier engine, carried out under contract No. AF-04(645)-4.

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NOMENCLATURE

- $A$  = cross sectional area of calorimeter
- $c$  = specific heat of material
- $c_p$  = specific heat of gases at constant pressure
- $D$  = body diameter
- $D_e$  = diameter of nozzle exit
- $g$  = gravitational constant
- $h$  = enthalpy
- $k$  = conductivity of material
- $k$  = ratio of densities across a normal shock
- $p$  = pressure
- $q$  = heat transfer rate
- $R$  = universal gas constant
- $r$  = radius of body
- $r_s$  = outside radius of shell
- $r_{in}$  = inside radius of shell
- $T$  = temperature
- $t$  = thickness
- $t_{eq}$  = equivalent thickness defined by Eqs. (37) or (38)
- $U_e$  = velocity at the outer edge of the boundary layer
- $w$  = weight of calorimeter
- $x$  = distance along centerline measured from nozzle exit (Section III)
- $x$  = distance along surface of body (Section I)

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Greek Symbols $\alpha = k/\rho c$  thermal diffusivity of material $\alpha(T_s, T_w) =$  function given by Eq. (8) $\alpha' =$  function given by Eq. (14) $\alpha'' =$  function given by Eq. (17) $\gamma =$  ratio of specific heats $\theta =$  angle between radius vector and centerline of body $\kappa =$  constant in the temperature-viscosity relation $\mu =$  viscosity in lb/sec. ft $\Xi =$  angle about the axis of body $\rho =$  density of gas or material $\tau =$  time in sec. $\chi = q_{c120}/q_{t120} =$  fraction of heat losses by conduction to total heat input at  $T_w = 620^\circ R$ Subscripts $c =$  chamber $l =$  conduction losses $s =$  at the stagnation point, also stagnation conditions, unless otherwise noted. $t =$  total $w =$  wall conditions $\infty =$  conditions ahead of shock $*$  = evaluated at a reference temperature

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SUMMARY

A review of the available theoretical expressions on the velocity gradient at the stagnation point of a cylinder or sphere, (an important parameter in stagnation point heat transfer) discloses no appreciable discrepancy between the various theories. The equation of Fay and Riddell, derived for the case of dissociated air, is next transformed by using the perfect gas law and the available data on the viscosity-temperature relation of the exhaust gases of the vernier engine. A simplified equation giving the heat transfer rate as a function of the body diameter, the Mach number ahead of the shock, and the stagnation or chamber pressure is thus obtained.

As a test of the validity of this procedure, the incompressible flow equation given by Sibulkin is modified by the incorporation of the reference temperature method of Eckert. This is compared with the previous result. The maximum deviation of this result from that previously determined is found to be less than 4 percent.

In Section II, two principal sources of error in the experimental data, the error due to the curvature and thickness of the calorimeters and that due to conduction losses, are discussed, and a method for the elimination of these errors is given. It is also shown here that the discrepancy between theory and experiment, with regard to the variation of the heat transfer rate with wall temperature, can be explained easily when the conduction losses are taken into account.

In Section III a comparison is made between the theoretical and experimental results. It is found that the adjusted experimental values are in good agreement with the theoretical curve, while the unadjusted data deviates appreciably in the region of low heat transfer (high Mach number) of the jet, where the conduction losses become important.

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I THEORETICAL HEAT TRANSFER RATES1 - Velocity Gradient at the Stagnation Point of a Blunt Body

The velocity gradient at the outer edge of the boundary layer near the stagnation point is required in order to compute the heat transfer rate as shown in Section 2 below. A number of available theoretical treatments give approximately the same value for the velocity gradient.

L. Lees and Fay & Riddell (Reference 1) use the hypersonic or Newtonian flow approximation which leads to the expression

$$\left(\frac{du_e}{dx}\right)_s = \frac{2}{D} \sqrt{\frac{2p}{\rho_s}} \quad (1)$$

Using the perfect gas law this equation takes the form

$$\left(\frac{du_e}{dx}\right)_s = \frac{2}{D} \sqrt{2RT_s} \quad (1a)$$

Hayes (as reported in Reference 2) gives an expression for the velocity gradient in terms of  $k$ , the density ratio across the normal shock:

$$\left(\frac{du_e}{dx}\right)_s = \frac{2u_\infty}{D} \sqrt{2k} \quad (2)$$

Using the definition of the speed of sound and expressing  $k$  in terms of the free stream Mach number one obtains, after some manipulation,

$$\left(\frac{du_e}{dx}\right)_s = \frac{2}{D} \sqrt{\frac{2\gamma}{\gamma+1}} \sqrt{2RT_s} \quad (2a)$$

Finally, Li and Geiger (Reference 2) give a modified expression

$$\left(\frac{du_e}{dx}\right)_s = \frac{2u_\infty}{D} \sqrt{k(2-k)} \quad (3)$$

After some manipulation this equation can be put in the following form:

$$\left(\frac{du_e}{dx}\right)_s = \frac{2}{D} \sqrt{\frac{2\gamma}{\gamma+1} \left[ 1 - \frac{\gamma-1}{2(\gamma+1)} - \frac{1}{(\gamma+1) M_\infty^2} \right]} \sqrt{2RT_s} \quad (3a)$$

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Eqs (2a) and (3a) are of the same form as Eq. (1a) except for a small factor which for  $\gamma = 1.267$  is equal to 1.055 in the case of Eq. (2a), and 1.015 in the case of Eq. (3a) with  $M=5$ . These equations are plotted in Figure 1. Since the velocity gradient appears as a square root term in the heat transfer equation, the discrepancy in using any one of the above equations will be small.

## 2 - Simplified Heat Transfer Equation for a Perfect Gas

Fay and Riddell (Ref. 1) give an equation for heat flux to the stagnation point of a body of revolution in dissociated air which, for Prandtl number equal to 0.71, is

$$q = 0.94 (\rho_w \mu_w)^{0.1} (\rho_s \mu_s)^{0.4} (h_s - h_w) \sqrt{\left(\frac{du}{dx}\right)_s} \quad (4)$$

Recently, Beckwith pointed out that the above equation also correlates the results of perfect-gas solutions, provided that Sutherlands formula for viscosity is used.

Equation (4) takes the following form, after multiplication and division by  $(\rho_s \mu_s)^{0.1}$  and substitution of Eq. (1a)

$$q = 0.94 \frac{\sqrt{2}}{\sqrt{D}} \left(\frac{\rho_w \mu_w}{\rho_s \mu_s}\right)^{0.1} \sqrt{\rho_s \mu_s} (h_s - h_w) (2gRT_s)^{1/4} \quad (5)$$

Using the perfect gas law and considering that the pressure,  $p$ , is constant across, and at the outer edge of the boundary layer, one obtains

$$q = \frac{1.33}{\sqrt{D}} \left(\frac{\mu_w T_s}{\mu_s T_w}\right)^{0.1} \sqrt{\frac{\mu_s}{RT_s}} \sqrt{p} (C_{p_s} T_s - C_{p_w} T_w) (2gRT_s)^{1/4} \quad (6)$$

or, grouping all the terms that depend on the temperatures,  $T_s$  and  $T_w$ ,

$$q = \alpha(T_s, T_w) \frac{\sqrt{p}}{\sqrt{D}} \quad (7)$$

where

$$\alpha(T_s, T_w) = 1.33 \left(\frac{\mu_w T_s}{\mu_s T_w}\right)^{0.1} \sqrt{\frac{\mu_s}{RT_s}} (C_{p_s} T_s - C_{p_w} T_w) (2gRT_s)^{1/4} \quad (8)$$

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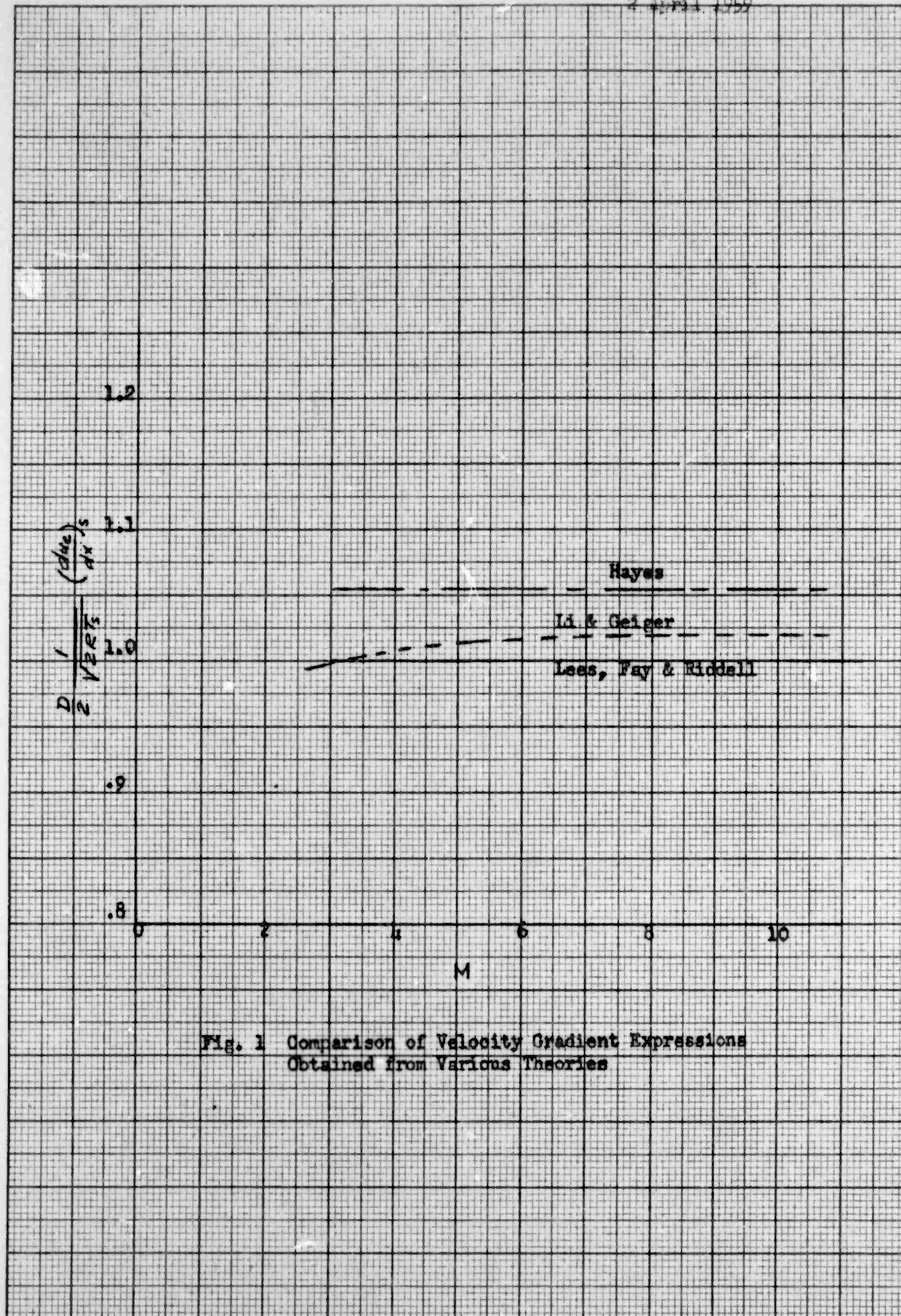


Fig. 1 Comparison of Velocity Gradient Expressions  
Obtained from Various Theories



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For hypersonic flow the value of  $p$  is given approximately by

$$p \approx \gamma p_\infty M_\infty^2 \quad (9)$$

where, from the one-dimensional flow relations

$$p_\infty = \frac{p_c}{\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\gamma/(\gamma-1)}} \quad (10)$$

therefore,

$$p \approx \frac{\gamma p_c M_\infty^2}{\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\gamma/(\gamma-1)}} \quad (11)$$

Eq. (7) then becomes, after substitution of Eq. (11),

$$q = \frac{\alpha \sqrt{\gamma p_c} M_\infty}{\sqrt{D} \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\gamma/2(\gamma-1)}} \frac{Btu}{sec. ft^2} \quad (12)$$

or, if  $p_c$  is given in  $lbs/in^2$ , and  $D$ , in inches

$$q = \frac{\alpha' \sqrt{\gamma p_c} M_\infty}{\sqrt{D} \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\gamma/2(\gamma-1)}} \frac{Btu}{hr ft^2} \quad (13)$$

where

$$\alpha' = 1.492 \times 10^5 \alpha \quad (14)$$

For  $\gamma=1.267$  and  $p_c = 347$  psia,  $q$  becomes a function of Mach number and diameter only.

$$q = \frac{\alpha''}{\sqrt{D}} \frac{M_\infty}{\left(1 + 0.1335 M_\infty^2\right)^{2.37}}, (D \text{ inches}) \quad (15)$$

where

$$\begin{aligned} \alpha'' &= 3.13 \times 10^6 \alpha \\ &= 4.16 \times 10^6 \left(\frac{\mu_w T_s}{\mu_s T_w}\right)^{0.1} \sqrt{\frac{\mu_s}{RT_s}} (c_{ps} T_s - c_{pw} T_w) (2gRT_s)^{1/4} \end{aligned} \quad (16)$$

in a slightly different form

$$\alpha'' = 1.13 \times 10^7 \left(\frac{\mu_w T_s}{\mu_s T_w}\right)^{0.1} \frac{\sqrt{\mu_s}}{(RT_s)^{1/4}} (c_{ps} T_s - c_{pw} T_w) \quad (17)$$

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Eq. (17) is plotted in Fig. 2 for a range of values of  $T_w$  using  $T_s$  as a parameter, and the available values of viscosity and specific heat of the exhaust gases. It is noted from this equation that the effect of wall temperature on the heat transfer rate is contained in the temperature gradient term, the term  $(\mu_w T_s / \mu_s T_w)^{0.1}$  being insensitive to a variation of  $T_w$  of a few hundred degrees. Thus, in the range of  $T_w = 600 - 900^\circ R$  and  $T_s = 5000 - 6000^\circ R$ , Eq. (17) can be simplified to the following equation, accurate to  $\pm 1.5$  percent:

$$\alpha'' = 3.65 \times 10^3 (c_{p_s} T_s - c_{p_w} T_w) \quad (18)$$

and the heat transfer rate, from Eq. (15), becomes

$$q = \frac{3.65 \times 10^3}{\sqrt{D}} \frac{M_\infty}{(1 + 0.1335 M_\infty^2)^{2.37}} (c_{p_s} T_s - c_{p_w} T_w), \quad \left| \begin{array}{l} p_e = 347 \text{ psia} \\ T_w = 600 - 900^\circ R \\ T_s = 5000 - 6000^\circ R \\ D \text{ in inches} \end{array} \right| \quad (19)$$

For  $T_s = 5500^\circ R$ ,  $T_w = 620^\circ R$ , and using the corresponding values for  $\mu$  and  $c_p$ , one finds  $\alpha'' = 9.55 \times 10^6$ , so that

$$q = \frac{9.55 \times 10^6}{\sqrt{D}} \frac{M_\infty}{(1 + 0.1335 M_\infty^2)^{2.37}} \quad \left| \begin{array}{l} p_e = 347 \text{ psia} \\ T_s = 5500^\circ R \\ T_w = 620^\circ R \\ D \text{ in inches} \end{array} \right| \quad (20)$$

The above equations apply to the case of a sphere. For the case of a cylinder, the equivalent equations are obtained by dividing by  $\sqrt{2}$ .

### 3 - Comparison with the Equation Obtained by the Reference Temperature Method.

It has been suggested that the incompressible stagnation point heat transfer equation given by Sibulkin can be used for compressible flow provided that the gas properties are evaluated at a reference temperature given by Eckert. (See Ref. 3). Thus, the modified equation is

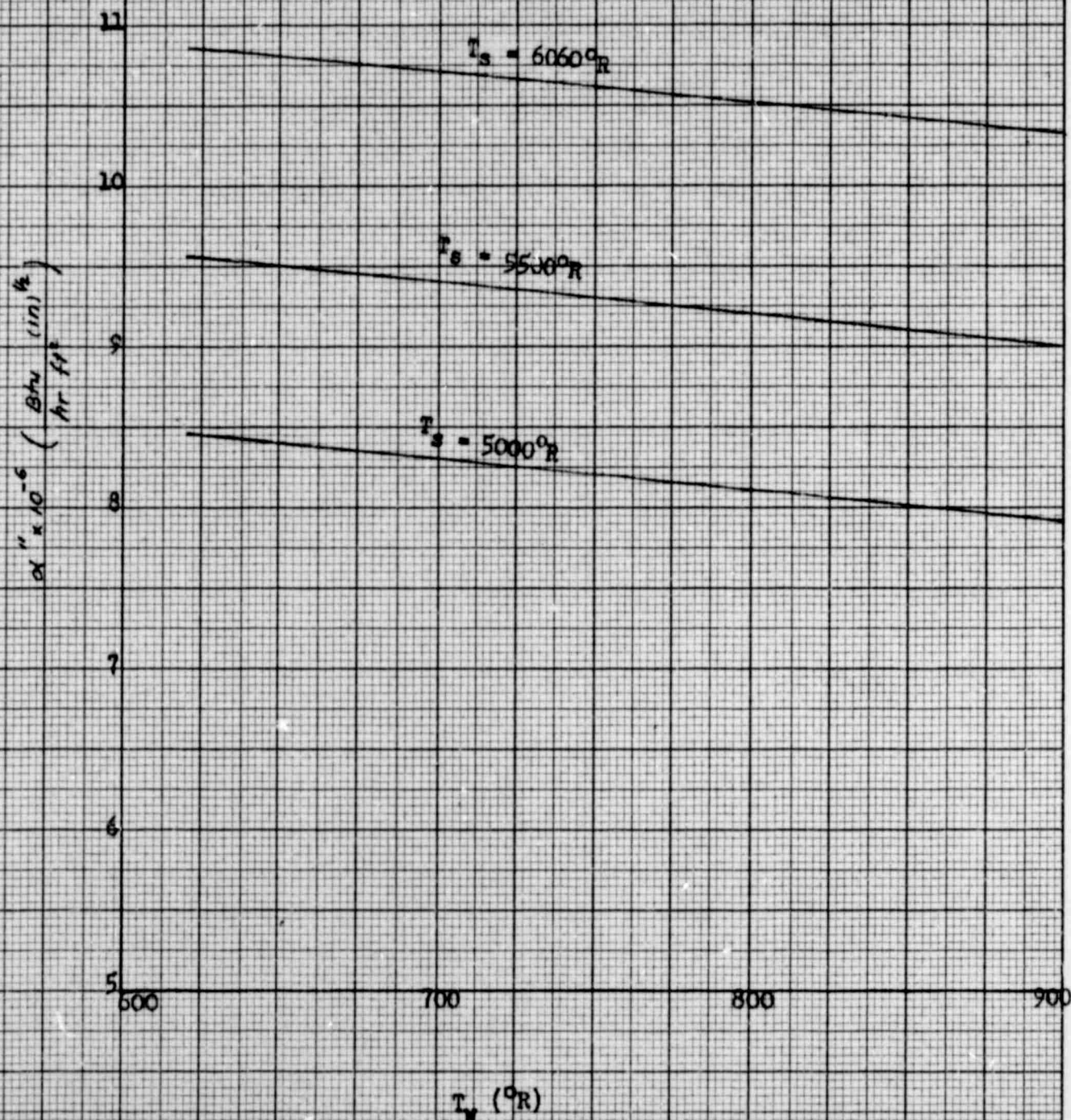
$$q_{nt} = \frac{0.763}{(Pr)^{0.6}} \sqrt{\rho^* \mu^*} (h_s - h_w) \sqrt{(du_e/dx)_s} \quad (21)$$

which for  $Pr = 0.71$  becomes

$$q_{nt} = 0.935 \sqrt{\rho^* \mu^*} (h_s - h_w) \sqrt{(du_e/dx)_s} \quad (21a)$$



Fig. 2 Values of the Function  $\alpha''$  for Various Stagnation and Wall Temperatures (Plot of Eq. 17)



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where the reference temperature is computed from the simplified equation

$$T^* = \frac{1}{2} (T_w + T_s) \quad (22)$$

Comparison of Eqs. (21a) and (4) gives

$$\frac{q_{r.t.}}{q} = \frac{\sqrt{\rho^* \mu^*}}{\left(\frac{\rho_w \mu_w}{\rho_s \mu_s}\right)^{0.1} \sqrt{\rho_s \mu_s}} \quad (23)$$

Using the perfect gas relation,  $p = \rho RT$ , Eq. (23) may be modified as follows:

$$\begin{aligned} \frac{q_{r.t.}}{q} &= \frac{\sqrt{\frac{p \mu^*}{RT^*}}}{\left(\frac{\mu_w T_s}{\mu_s T_w}\right)^{0.1} \sqrt{\frac{p \mu_s}{RT_s}}} \\ &= \frac{\sqrt{\frac{\mu^*}{\mu_s} \frac{T_s}{T^*}}}{\left(\frac{\mu_w T_s}{\mu_s T_w}\right)^{0.1}} \end{aligned} \quad (24)$$

A temperature - viscosity relation of the form

$$\frac{\mu^*}{\mu_w} = X \frac{T^*}{T_w} \quad (25)$$

may be assumed, and Eq. (24) becomes

$$\begin{aligned} \frac{q_{r.t.}}{q} &= \frac{\sqrt{X \frac{\mu_w}{\mu_s} \frac{T_s}{T_w}}}{\left(\frac{\mu_w T_s}{\mu_s T_w}\right)^{0.1}} \\ &= X^{1/2} \left(\frac{\mu_w}{\mu_s} \frac{T_s}{T_w}\right)^{0.4} \end{aligned} \quad (26)$$

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A value of  $N = 0.62$  to  $0.68$  was obtained from the viscosity temperature data of the exhaust gases in the range of  $T_w = 620 - 900^\circ R$ . Using this value of  $N$  in Eq. (26) the ratio of the heat transfer rates given by the two equations was found to vary slightly from unity

$$\frac{q_{rt}}{q} = 1.025 \text{ to } 1.038$$

Thus Eqs. (21a) and (4) give approximately the same heat transfer rate. Values obtained from Eq. (21) have been plotted in Figure (5).

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## II ANALYSIS OF THE EXPERIMENTAL RESULTS

### 1 - General Equation of Heat Conduction in the Calorimeter

The differential equation for unsteady-state heat conduction in spherical coordinates is

$$\frac{\partial T}{\partial \tau} = \alpha \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 T}{\partial \xi^2} \right] \quad (27)$$

Assuming that all derivatives of  $T$  with respect to  $\theta$  and  $\xi$  are not functions of  $r$ , integrating with respect to  $r$ , and using the boundary condition  $\frac{\partial T}{\partial r} = 0$  at  $r = r_{in}$  one obtains the temperature gradient at the outer surface,  $r = r_s$ ,

$$\left( \frac{\partial T}{\partial r} \right)_s = \frac{r_s^3 - r_{in}^3}{3\alpha r_s^2} \frac{\partial T}{\partial \tau} - \frac{r_s - r_{in}}{r_s^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) \quad (28)$$

The heat transfer rate is then given by

$$[q_s]_{\text{sphere}} = -k \left( \frac{\partial T}{\partial r} \right)_s = -\rho c \frac{r_s^3 - r_{in}^3}{3r_s^2} \frac{\partial T}{\partial \tau} + k \frac{r_s - r_{in}}{r_s^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) \quad (29)$$

The corresponding equation for heat transfer to a cylinder is found in a similar manner starting from the differential equation in cylindrical coordinates

$$\frac{\partial T}{\partial \tau} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] \quad (30)$$

Thus

$$[q_s]_{\text{cyl.}} = -\rho c \frac{r_s^2 - r_{in}^2}{2r_s} \frac{\partial T}{\partial \tau} + \frac{k}{r_s} \frac{\partial^2 T}{\partial \theta^2} \ln \left( \frac{r_s}{r_{in}} \right) \quad (31)$$

The first term on the right hand side of Eqs. (29) and (31) represents the heat storage in the shell element and the second term, the conduction away from the element. When the size of the element is taken equal to the size of the calorimeter this second term represents the heat losses by conduction.

## 2 - Correction for the Thickness and Curvature of the Calorimeter

The equation used in computing the heat storage in the calorimeter from the experimental temperature-time curve is:

$$q_{st.exp.} = \frac{W}{A} C \frac{dT}{d\tau} \quad (32)$$

This equation applies strictly to the case of a flat plate. For a calorimeter having the form of a spherical shell the storage term in Eq. (29) applies:

$$q_{st.th.} = \rho C \frac{r_s^3 - r_{in}^3}{3r_s^2} \frac{dT}{d\tau} \quad (33)$$

or

$$q_{st.th.} = \rho C t_{eq} \frac{dT}{d\tau} \quad (33a)$$

where

$$t_{eq} = \frac{r_s^3 - r_{in}^3}{3r_s^2} \quad (34)$$

which can be put in the form

$$q_{st.th.} = \rho C t \frac{A}{A} \frac{t_{eq}}{t} \frac{dT}{d\tau} = \frac{W}{A} C \frac{dT}{d\tau} \left( \frac{t_{eq}}{t} \right) \quad (35)$$

Comparison of Eq. (35) and (32) gives

$$q_{st.th.} = \left( \frac{t_{eq}}{t} \right) q_{st.exp.} \quad (36)$$

Eq. (36) applies also to the case of a cylindrical shell element; however for a sphere, one finds, after some manipulation of Eq. (34),

$$\left[ \frac{t_{eq}}{t} \right]_{sph.} = 1 - \frac{t}{r_s} + \frac{1}{3} \frac{t^2}{r_s^2} \quad (37)$$

whereas, for a cylinder, using the storage term of Eq. (31),

$$\left[ \frac{t_{eq}}{t} \right]_{cyl.} = 1 - \frac{1}{2} \frac{t}{r_s} \quad (38)$$



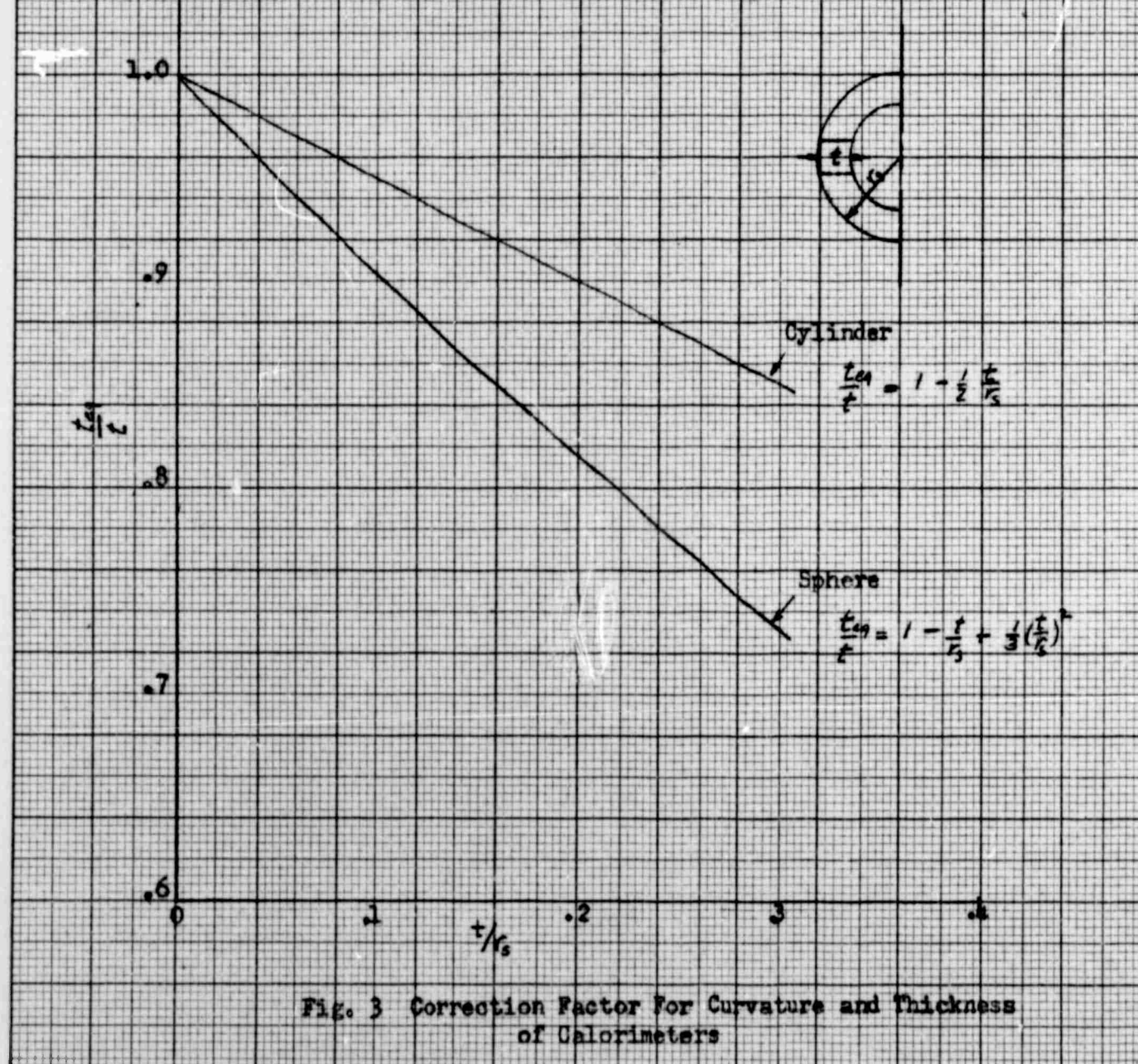


Fig. 3 Correction Factor For Curvature and Thickness of Calorimeters

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Eqs. (37) and (38) are plotted in Fig. 3. It is seen that for the values of  $\frac{r}{L} = 0.20$  or  $0.267$  of the test models ( $t = 0.1$ ,  $G = \frac{1}{2}$ " or  $\frac{3}{8}$ " ) the deviation from the flat plate equation, Eq. (32), is appreciable.

### 3 - Correction for Heat Losses by Conduction

To compute the losses by conduction a knowledge of the surface temperature distribution is required. Since no experimental values are available, a temperature distribution of the form

$$T = T_{ws} \cos \theta \quad (39)$$

may be assumed. This distribution checks well with experimental values obtained by the NACA at a Mach number of six, and for small values of  $\theta$  (For a calorimeter diameter of  $0.128$ " and a sphere diameter of  $3/4$ ",  $\theta$  is approximately  $9.6$  degrees) The temperature derivative, appearing in the conduction term of Eq. (29) is found from Eq. (39)

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{dT}{d\theta} \right) = -2 T_{ws} \cos \theta \approx -2 T_{ws} \quad (40)$$

and the heat loss, using the conduction term from Eq. (29), is

$$q_c = -2 k \frac{t}{G^2} T_{ws} \quad (\text{sphere}) \quad (41)$$

The above equation and the temperature distribution given by Eq. (39) are strictly valid for a homogeneous sphere. In the case of a calorimeter material different from the model material Eq. (41) can be used for estimating purposes, with the value of  $k$  for the sphere material. The validity of this assumption is demonstrated by an indirect approach in Section 4 below.

For the case of the cylinder.

$$\frac{d^2 T}{d\theta^2} = -T_{ws} \cos \theta \approx -T_{ws} \quad (42)$$

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and

$$q_l = -\frac{k}{r_s} T_{ws} \ln\left(\frac{r_s}{r_{in}}\right) \quad (\text{cylinder}) \quad (43)$$

4 - Effect of Conduction Losses on the Heat Transfer Rates at Different Wall Temperatures.

The large discrepancy between experimental results and theory with respect to the variation of the heat transfer rate with temperature, shown in Fig. 1, page 5 of Reference 4, can be accounted for by including the heat losses by conduction. The total heat transfer from the gas to the sphere at the stagnation point, for wall temperatures of 620°R and  $T_{ws}$  is respectively

$$q_{t620} = q_{exp620} + q_{l620} \quad (44)$$

$$q_{tT_{ws}} = q_{expT_{ws}} + q_{lT_{ws}} \quad (45)$$

From Eq. (44) it is seen that the heat losses are proportional to the temperature and therefore

$$q_{lT_{ws}} = \frac{T_{ws} - 460}{620 - 460} q_{l620} \quad (46)$$

Moreover from the theoretical heat transfer results

$$q_{lT_{ws}} = \frac{T_s - T_{ws}}{T_s - 620} q_{l620} \quad (47)$$

Combining Eqs. (44) and (45) substituting Eqs. (46) and (47) and rearranging one obtains

$$\frac{q_{expT_{ws}}}{q_{exp620}} = \frac{\frac{T_s - T_{ws}}{T_s - 620} - \frac{T_{ws} - 460}{160} \chi}{1 - \chi} \quad (48)$$

$$\chi = \frac{q_{l620}}{q_{t620}} \quad (49)$$



$$\frac{q_{exp} T_{w,620}}{q_{f,620}} = \frac{\left( \frac{T_s - T_{w,620}}{T_s - 620} \right) - \frac{T_{w,620} - 460}{160} \chi}{1 - \chi}$$

$$\chi = \frac{q_{f,620}}{q_{f,620}}$$

○ Experimental points

--- Experimental curve

obtained for  $\chi = 11, 3/4$  Dia sphere

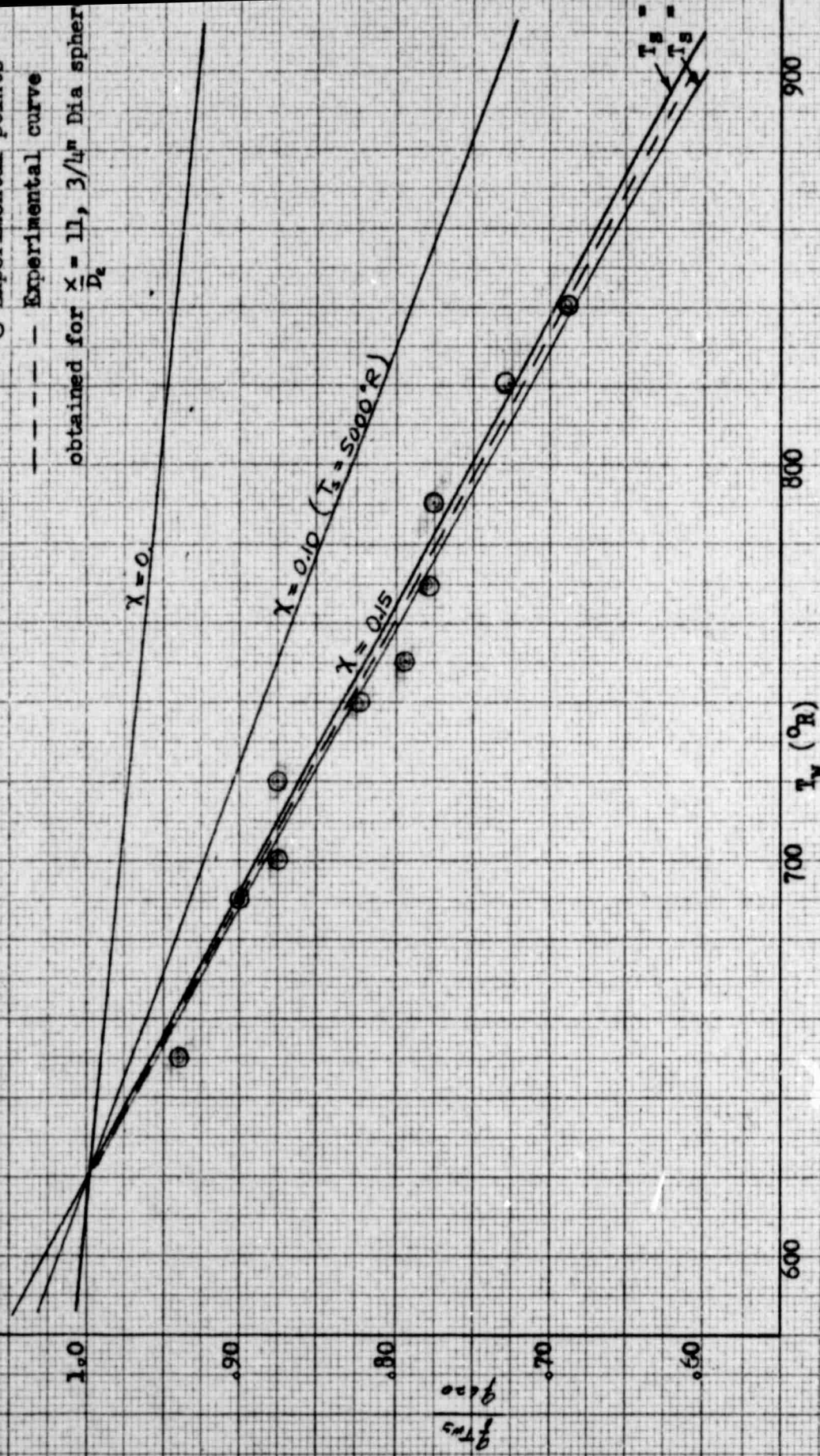


Fig. 4 Variation of the Heat Transfer Rate  
with Wall Temperature

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Eq. (48) plotted in Fig. 4 for  $\chi = 0, 0.10, 0.15$

To determine the value of  $\chi$  for the conditions of Fig. 1 of the above mentioned report, the heat loss was computed from Eq. (41) to be  $0.71 \times 10^5$  Btu/hr ft<sup>2</sup> and  $q_r$  was found from theory at  $x/D_c = 11$  to be  $4.6 \times 10^5$  so that  $\chi \approx 0.15$ . The experimental curve is also plotted in Fig. 4 as a dashed line. It coincides with the theoretical curve for  $\chi = 0.15$ , which is the same as the computed value. This agreement indicates that the use of Eq. (41) for computing losses is justified. The use of the conductivity of the sphere material (steel) may also be justified from the fact that the conductivity of the calorimeter material (copper) is much larger.

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III CORRELATION OF THEORY AND EXPERIMENT

The experimental results reported in Reference (7) must be corrected for the errors discussed in Sections 2 and 3. Using  $q_{exp 620}$  to denote the data as reported, the adjusted heat transfer rate is given by.

$$q_{adj} = \left(\frac{t_q}{t}\right) q_{exp 620} + q_{620} \quad (50)$$

For a 3/4 in-diameter sphere with shell thickness  $t=0.10$  in. this equation becomes

$$q_{adj} = 0.76 q_{exp 620} + 71,000 \quad (50a)$$

The experimental points, thus adjusted, are plotted in Fig. 5. The data on models with insulated calorimeters has been adjusted only for the error due to curvature. The simplified equation, Eq. (20) in Section I, has also been plotted in this figure using the variation of  $M$  with  $\sqrt{D_c}$  (See Ref. 5) shown in the lower portion of the curve. The agreement between theory and experiment is seen to be satisfactory. The experimental points have also been plotted in Fig. 6 without any correction and compared to the theoretical results. It is seen that heat losses cause an appreciable error at the high-Mach-number portion of the curve where the heat input is low. The test results on a 3/4" cylinder are plotted in Figure 7 along with a plot of values of  $q$  obtained from Eq. (20) divided by  $\sqrt{2}$ . The experimental points are plotted as reported in Reference (6) without any correction for the curvature and thickness of calorimeter and/or for conduction losses. From Fig. 3  $t_q/t$  is approximately equal to 0.87, for a cylindrical shell element. However, the calorimeter used was circular and this correction cannot be made with certainty. Moreover a uniform constant was used for all calorimeters in reducing the experimental data, even though this constant was expected to vary slightly with each calorimeter. The heat loss computed from Eq. (43) at a wall temperature of 620°R is 41,000 Btu/hr ft<sup>2</sup>. This correction will raise the values of the experimental points at high Mach numbers, and give a better agreement with the theoretical curve. It is emphasized that corrections for curvature and heat conduction have not been applied to the data presented in Fig. 7.

Finally, it should be pointed out that, in view of the appreciable rarefaction of the jet at a Mach number of 10 or higher (See Fig. A-2 p. 46 of Reference 8), slip-flow effects will tend to reduce the heat transfer rates to cylinders and spheres, and the theoretical curve is not strictly applicable beyond this Mach number. From Fig. D-4 (p. 71) of this reference it is estimated that there will be a 15 percent reduction in the average heat

$q \frac{Btu}{hr ft^2}$

$\Delta q = 0.76 q_{lim}$

$\odot q = 0.76 q_{exp} + 71.000$

x Theory, Reference Temperature Method

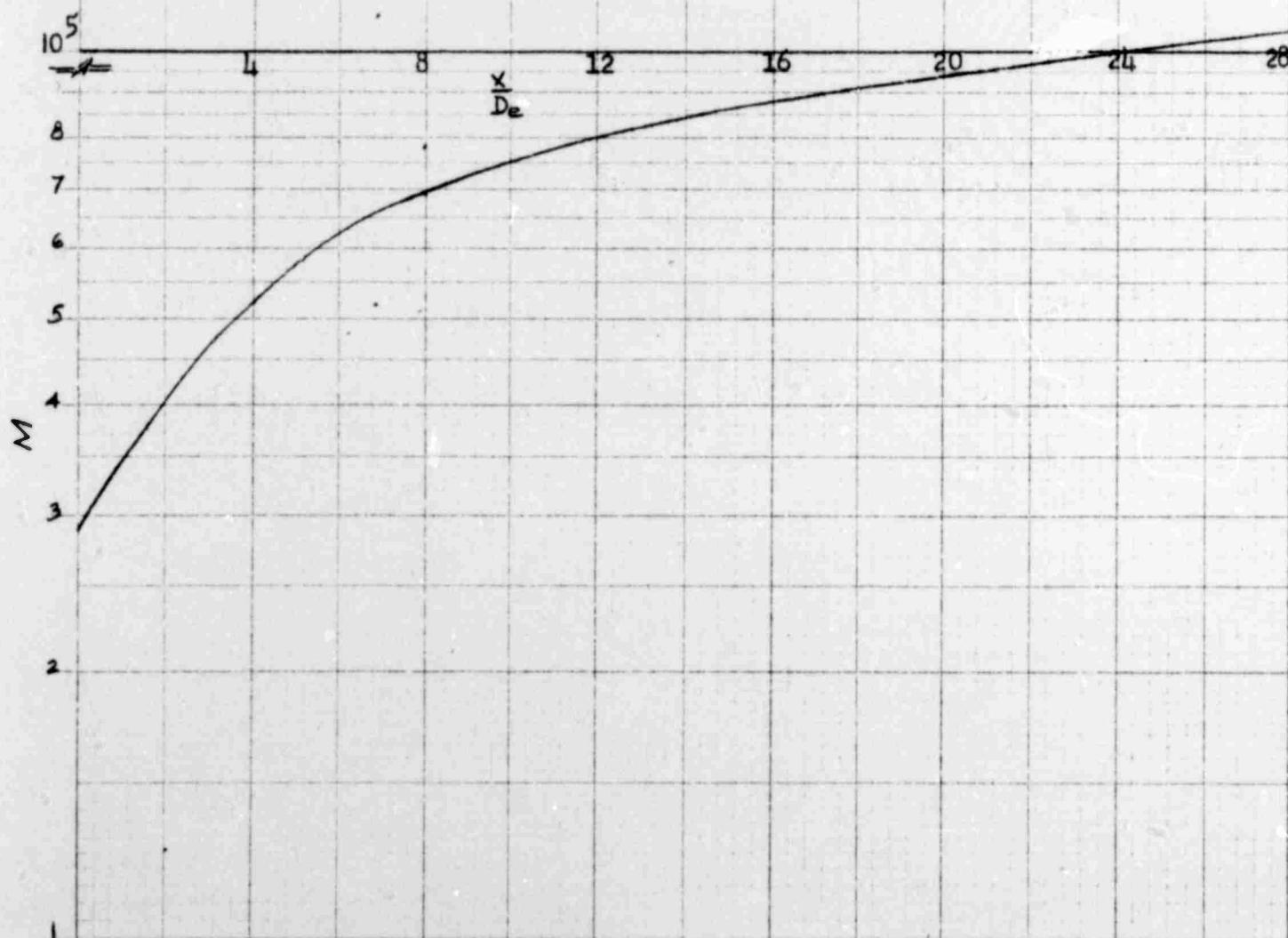
$q = \frac{9.55 \times 10^6}{\sqrt{D}} \frac{M}{(1 + .1335 M^2)^{2.57}}$

$D = \frac{3}{4}$  Sphere

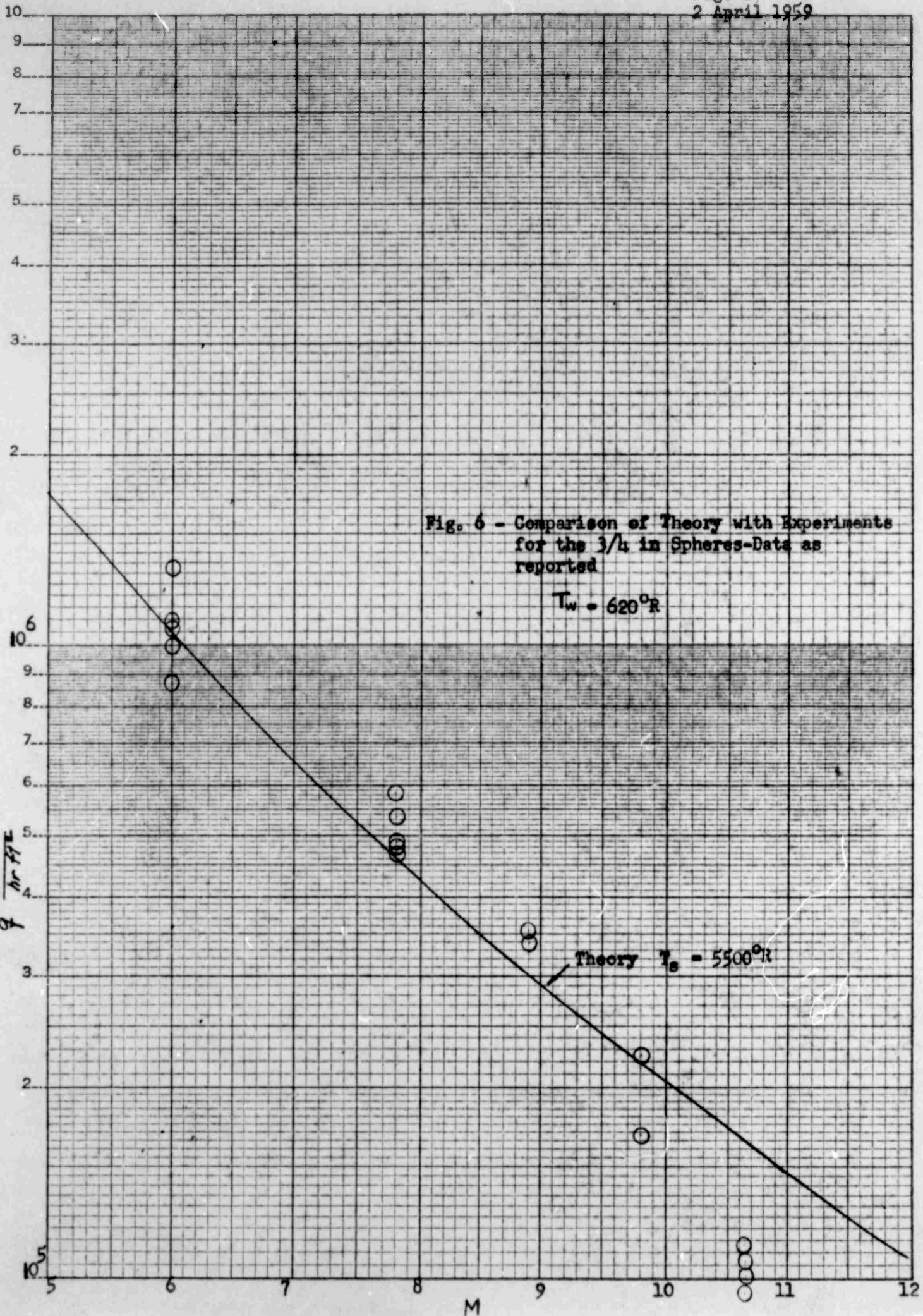
$T_w = 620^\circ R$

$T_s = 5500^\circ R$

Fig. 5 Comparison of Theory with the Adjusted Experimental Data for Spheres



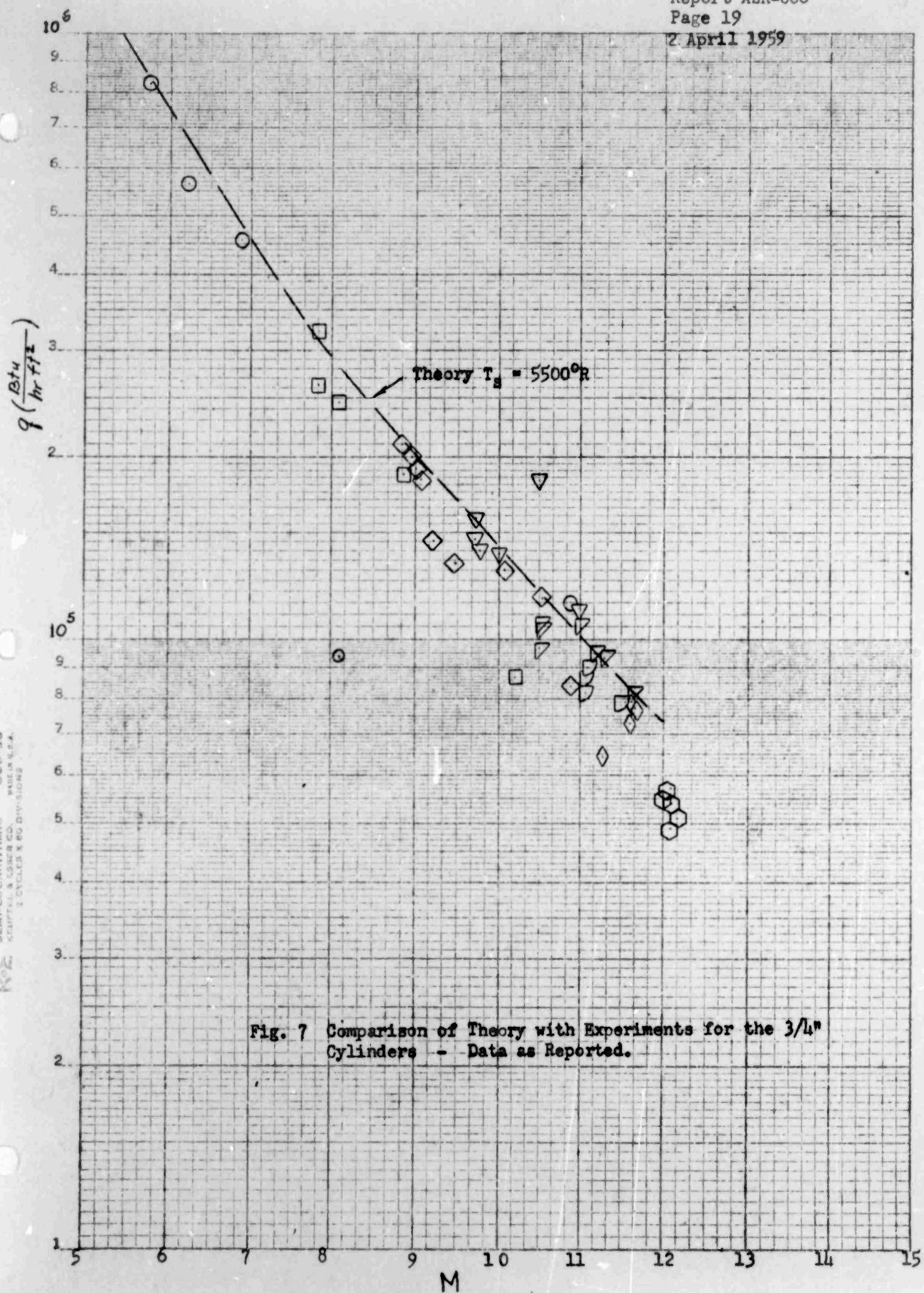




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$q$   $\frac{Btu}{hr ft^2}$

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transfer rate to a  $3/4$ " sphere at  $M = 10.5$ . Thus, the experimental results in this rarefied region should be expected to be somewhat lower than the theoretical curve, which was based on continuum-flow theory.

#### REFERENCES

- 1 - Fay, J. A. and Riddell, F. R., "Theory of Stagnation Point Heat Transfer in Dissociated Air", J.A.S. vol 25, No. 2, pp 73-85, February, 1958.
- 2 - Li, T. Y. and Geiger, R., "Stagnation Point of a Blunt Body in Hypersonic Flow", J.A.S vol 24, pp 25-32, January, 1957.
- 3 - Romig, M. F., "Stagnation Point Heat Transfer for Hypersonic Flow", Jet Propulsion, vol 26, No. 12, pp 1098, December, 1956.
- 4 - Cruse, R. E., "Heating Rates to Spheres in a 1/10 Scale Atlas Vernier Rocket Engine Exhaust", Convair San Diego, Confidential Report RT58-0633, September, 1958.
- 5 - Bowyer, J. M., "Determination of the Envelopes and the Lines of Constant Mach Number for an Axially Symmetric Jet", Convair-Astronautics, Confidential Report No. ZJ-7-054, March, 1958.
- 6 - Cruse, R. E., "Heating Rates to Cylinders and Flat Plates in a 1/10 Scale Atlas Vernier Rocket Engine Exhaust" Convair, San Diego, Confidential Report RT 58-063, September, 1958
- 7 - Sergeant, R., "Further Investigation of Heating Rates to Spherical and Cylindrical Bodies in a 1/10 Scale Atlas Vernier Rocket Engine Exhaust", Convair, San Diego, Confidential Report RT 59-003, March 1959.
- 8 - Vasiliu, J., "Heat Transfer Rates to Plates, Cylinders, and Spheres in High-Speed, Rarefied-Gas Flow", Convair-Astronautics, Report No. ZJ-7-058TN, November, 1958.